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Techniques to Improve Computation of Zernike Polynomials

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Abstract: Zernike polynomials are widely used in adaptive optics (AO) and aberration studies including in vision optics. Estimation of wavefront usually involves computing the Zernike moments [1] and the results maybe used in, deformable mirrors for AO applications, scaled to different pupil sizes in case of VO, etc. Typical challenges in using Zernike polynomials include (a) the speed of computing the Zernike moments can be limiting for realtime applications, (b) the computation of higher-order Zernike polynomials can be slow, and precision limited. This paper presents some solutions to these problems in the form of classic computer science techniques of dynamic programming[5] and insight into computing higher order fields involving Zernike polynomials.

1.Introduction

Zernike polynomials are widely used in adaptive optics, vision optics and studies of optical systems for purposes of phase modeling, wavefront estimation, and optical field descriptions. We present some novel ideas to improve the computation speed of Zernike moments and extend their accuracy to higher orders. The motivation for this work comes from the insights gained working on Laguerre-Gaussian and Hermite-Gaussian mode superposition field synthesis and moment decompositions in the study [3].

Zernike polynomials of order $\underline{n}$ and azimuthal index m is represented below following notation of [2].

$$Z_n^m(\rho,\theta) = \begin{cases} N_n^m R_n^m(\rho)\cos(m\theta) & \text{for } m \geq 0 \\ -N_n^m R_n^m(\rho)\sin(m\theta) & \text{for } m < 0 \end{cases}, \quad |m| \leq n,\ n-|m| \text{ even}, \tag{1}$$

where the radial polynomial $R_n^m(\rho)$ is given by

$$R_n^m(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s (n-s)!}{s!\,[(n+|m|)/2-s]!\,[(n-|m|)/2-s]!}\rho^{n-2s}, \tag{2}$$

and N_n^m is the normalization factor given by

$$N_n^m = \sqrt{\frac{2(n+1)}{1+\delta_{m,0}}}. \tag{3}$$

The optical fields are represented as a superposition sum of eq. (1) to describe their phase structure at particular optical frequency.

2. Techniques

2.1 Dynamic Programming

For the purpose of this paper we use the reference code for Zernike Polynomials [4] but rewrite in Numpy Python to leverage the caching utility and free availability unlike MATLAB.

Dynamic programming (DP) [5] provides a methodology to reduce computation of partial solutions (sub-problems) en-route to the main problem; for the purpose of the Zernike polynomials this is implicit due to 2-term recurrence relationship [1]. In this case DP is same as caching in the computation of the Zernike basis functions which can significantly speedup the computation of the matrices and fields shown in Table. 1:

Table 1. Effect of Caching on Computation of Zernike Polynomials

N	No Cache (s)	Cached
10	0.0936s	0.0936s
12	0.1707s	0.0755s
14	0.2752s	0.1009s
16	0.4025	0.1274s

2.2 Large Order Zernike Field Computation

For the large order Zernike field computation which is presented over *rho*, *theta*, transverse variables of the spatial coordinate; while typical implementations compute the Zernike polynomials repetitively for each of the spatial coordinate we suggest the alternative of promoting the 2-term recurrence of Zernike Polynomials into the recurrence of the field scaled by suitable coordinate.

We note the optimizations in 2.1 and 2.2 can be cumulatively used in any implementation providing a product of efficiency speedup.

3. Conclusion

We presented results to use some commonly known computer science ideas and an insight (2.2) to improve the use of Zernike polynomials; this work will be extended to other orthonormal family of polynomials forming modal basis functions relevant to optical systems like LG, HG modes, etc. [3].

4. References

[1] Born, M., Wolf, E., Bhatia, A. B., Clemmow, P. C., Gabor, D., Stokes, A. R., … Wilcock, W. L. (1999). *Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Light* (7th ed.). Cambridge: Cambridge University Press.
[2] Huazhong Shu, Limin Luo, Guoniu Han, and Jean-Louis Coatrieux, "General method to derive the relationship between two sets of Zernike coefficients corresponding to different aperture sizes," J. Opt. Soc. Am. A 23, 1960-1966 (2006).
[3] Annamalai, Muthiah, "Mode Structure Of A Noiseless Phase-sensitive Image Amplifier" (2011). Electrical Engineering Dissertations. 48.
[4] Evan Czako (2025). Quick Zernike polynomial creation and decomposition (https://github.com/EvanCzako/Zernike-Polynomials-MATLAB), GitHub. Retrieved June 24, 2025.
[5] Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, *Introduction to Algorithms,* 4th *ed.* MIT Press.